Mechanical Systems and Signal Processing (2003) **17**(1), 9–20
doi:10.1006/mssp.2002.1534, available online at http://www.idealibrary.com on IDEAL®

WORKING GROUP 1: GENERATION OF VALIDATED STRUCTURAL DYNAMIC MODELS—RESULTS OF A BENCHMARK STUDY UTILISING THE GARTEUR SM-AG19 TEST-BED

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(Received 28 March 2002, accepted after revisions 1 October 2002)

1. AIM OF THE STUDY

In practice, the validation of analytical structural dynamic models is mainly based on comparing experimental modal analysis results with analytical predictions. Despite the high sophistication of analytical [finite element (FE)] modelling, practical applications often reveal considerable discrepancies between analytical and test results. In recent years, significant effort has therefore been expended on the development of mathematical procedures for updating analytical mass and stiffness matrices using dynamic test data. The success of these methods is governed not only by the skill of the analyst to assume an appropriate initial analysis model but also the source and the location of the erroneous parameters to be corrected. In practical applications, the source and location of the errors can be manifold, resulting in non-unique updated models with all of them fulfilling the mathematical criterion of minimising the test/analysis discrepancies.

The aim of the present benchmark study defined within the European COST Action F3 on 'Structural Dynamics' was not only to compare the different computational model updating (CMU) procedures using a common test structure but also to see if the expected non-uniqueness of the results due to different computational methods, different structural idealisations and different parameter sets and, of course, different test data sets can be tolerated with regard to the intended utilisation. The following requirements for a validated model were defined:

(1) The model must be capable of predicting the experimental modal data and/or the frequency response functions (FRFs) within the active frequency range and within certain accuracy limits, of course. The term active frequency range is related to the frequency range used for CMU.

The above criterion represents a minimum requirement which does not yet say much about the prediction quality of the model. The prediction quality should therefore be checked using the following additional criteria:

(2) Prediction of the eigenfrequencies and modes beyond the active frequency range.

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(3) Prediction of the FRFs obtained from loading conditions other than those used for CMU.

(4) Prediction of the modal data and/or FRFs of a modified structure. The structural modification might consist of one or more added masses or of changed boundary conditions.

The participants were allowed to generate any initial FE model that they found suitable. The FE model with the lowest model order and best fulfilling the above criteria should be considered the optimum model.

2. BENCHMARK DATA

The benchmark structure was a laboratory structure built to simulate the dynamic behaviour of an aeroplane. The structure was initially built for a benchmark study on experimental modal analysis conducted by the Structures and Materials Action Group (SM-AG19) of the Group for Aeronautical Research and Technology in EURope (GARTEUR) [1–3]. The test-bed was designed and manufactured by ONERA, France. Figure 1 shows the test structure geometry and the location of the measured degrees of

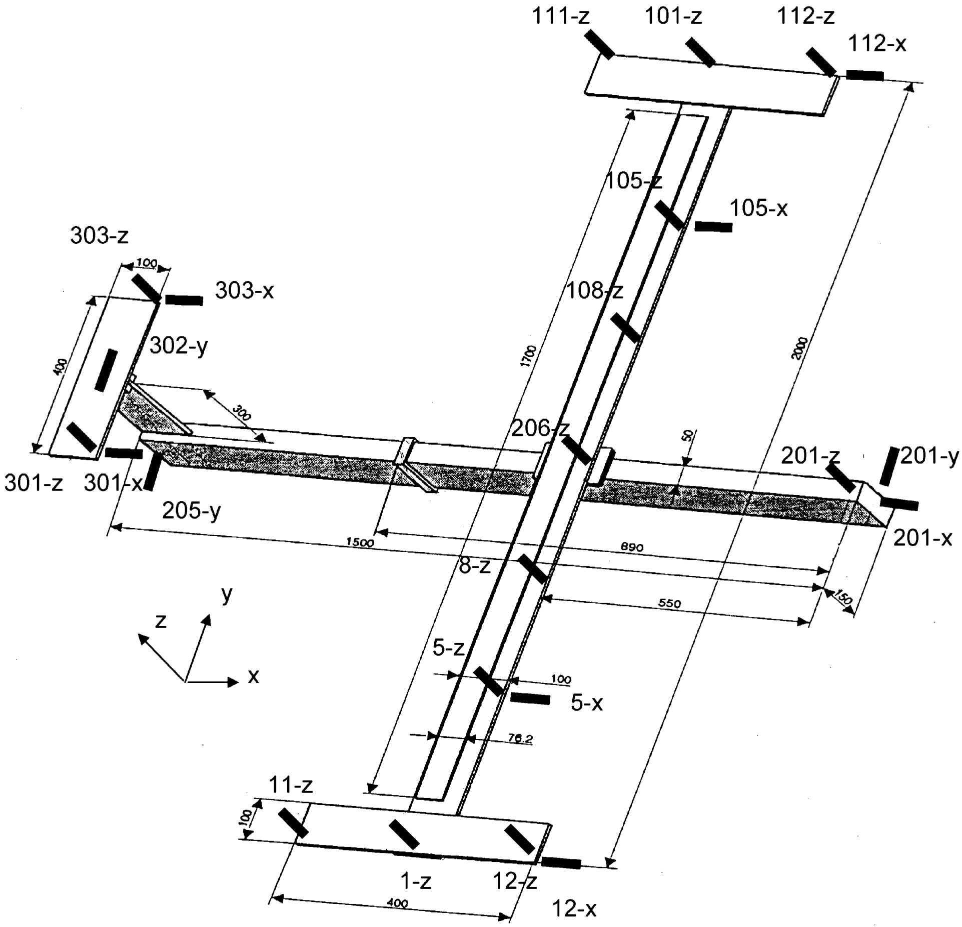

Figure 1. Geometry and accelerometer location.

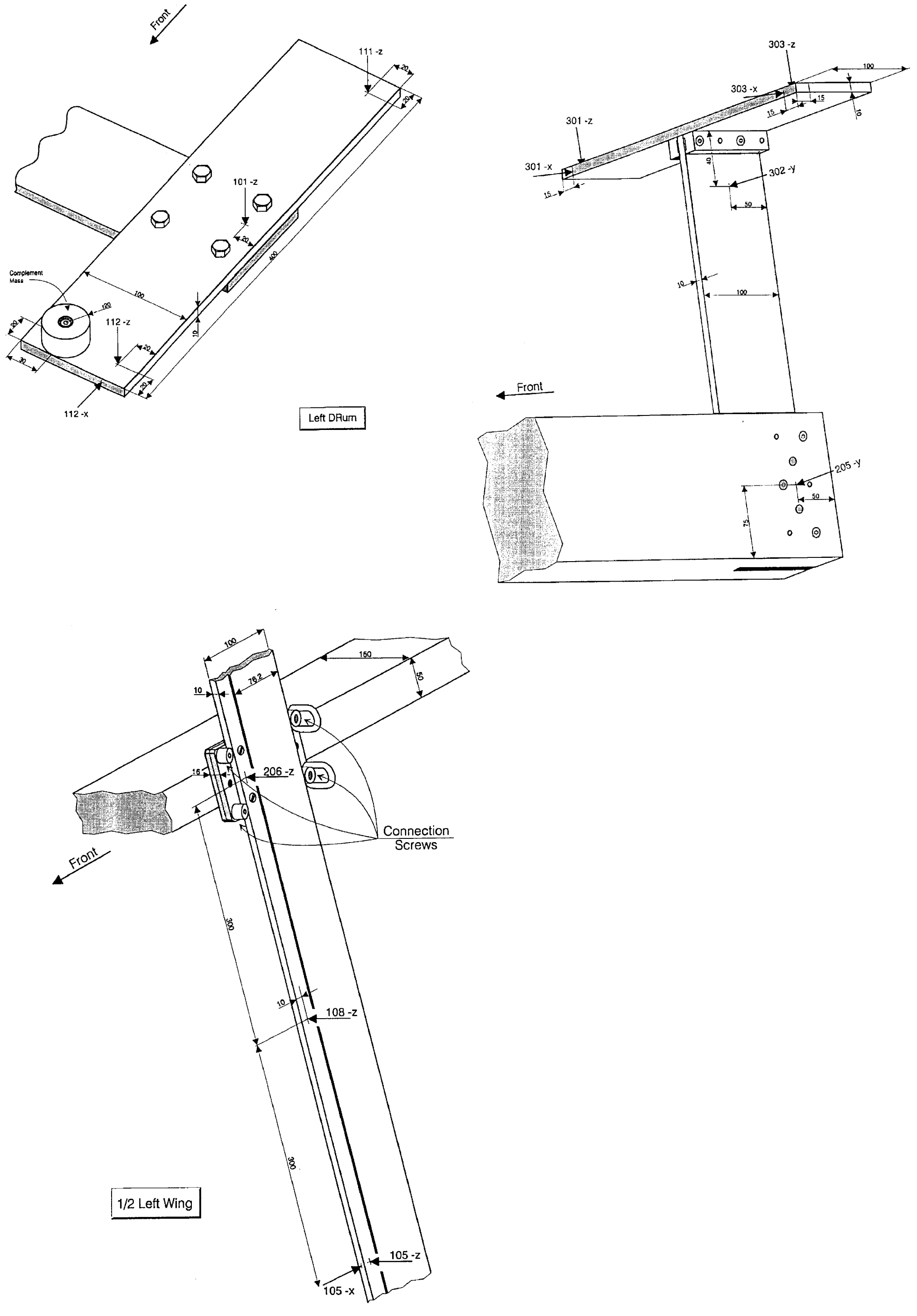

Figure 2. Design details.

freedom (dof). Structural details are given in Fig. 2. The overall length of the structure is 1.5 m, the wing span is 2.0 m and the overall mass is 44 kg. The material used was aluminium. In order to increase the damping, a $1.1 \times 76.2 \times 1700\,\text{mm}^3$ viscoelastic constraining layer was bonded to the wings. Further details are described in reference [1]. References [2–4] describe the experimental benchmark results for the modal data and the FRFs including their variability, whereas references [5–7] include some early model updating results.

For the present model validation benchmark the following test data sets were made available to the participants:

(a) Data from University of Manchester (U_MAN), UK: 24 FRFs due to excitation at the right wing tip (dof 12*z*) at 4096 frequency points up to about 80 Hz (Fig. 3 shows three examples).

(b) Data from DLR, Göttingen, Germany: The modal data between 6.376 Hz (Mode 1) and 151.317 Hz (mode 14), measured at 24 dofs using the phase resonance (PhR) technique.

(c) Data from re-testing the unmodified structure. About 5 years after the measurements described under (a) and (b) above, the Imperial College of Science, Technology and Medicine, London (IC), obtained the original structure from ONERA which allowed the original structure to be re-tested but also allowed structural modifications to be made. The data from the unmodified structure consisted of 24 FRFs due to excitation at the right wing tip (dof 12*z*) at 801 frequency points up to about 100 Hz [Fig. 3 shows three examples in comparison with the FRFs of (a)]. The modal data were extracted using the phase separation technique.

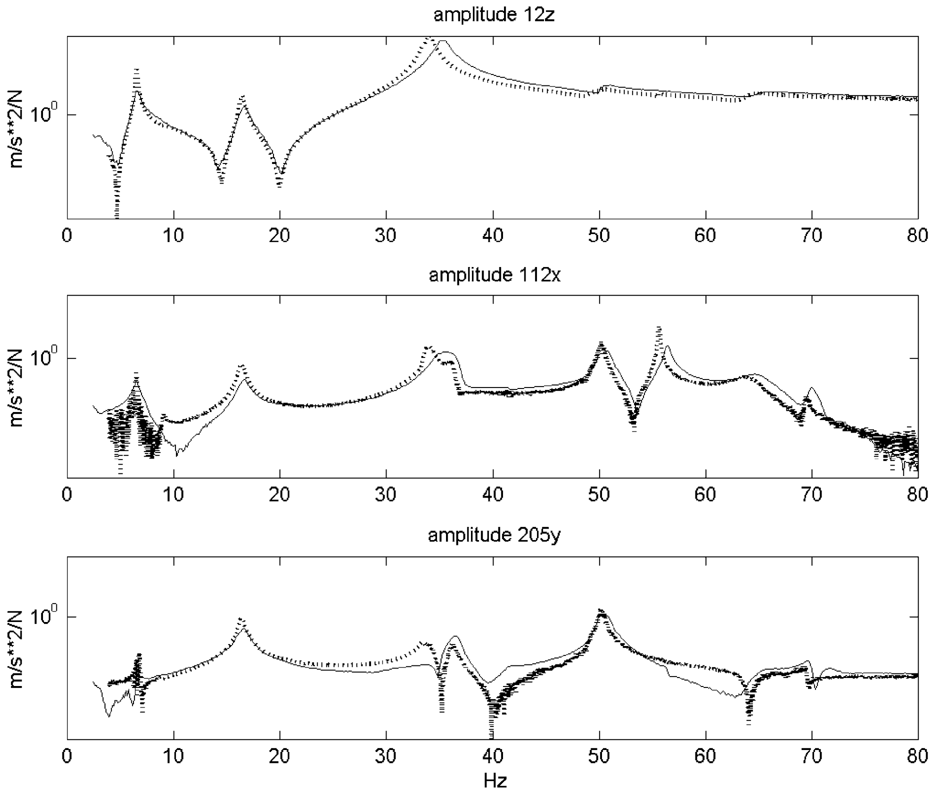

Figure 3. Frequency response functions of unmodified structure measured by IC/UWS —— and U_MAN- - - - -.

(d) Data of the modified structure measured by the IC and the University of Wales, Swansea (UWS), U.K.

(1) Modification 1 (IC) with 0.92 kg mass added at the tail: 24 FRFs due to excitation at the right wing tip (dof 12*z*) at 801 frequency points up to about 100 Hz.The modal data were also extracted using the phase separation technique.

(2) Modification 2 (UWS) with the existing 0.15 kg mass at the wing tip replaced by a 0.724 kg mass: 24 FRFs due to excitation at the right wing tip (dof 12*z*) at 801 frequency points up to about 200 Hz.The modal data were also extracted using the phase separation technique.

Figure 4 shows the mode shapes measured by DLR on the original unmodified structure by pure mode excitation (the PhR technique). To give an impression of the variability in the test data, Table 1 shows a comparison of the experimental modal data obtained from the DLR/PhR test and the modal data extracted from the FRFs of the U_MAN by a phase separation technique. The comparison shows that the correlation of the mode shapes expressed by the modal assurance criterion (*MAC*) is very good whereas the eigenfrequencies show a systematic shift. In practical applications, the test data variability

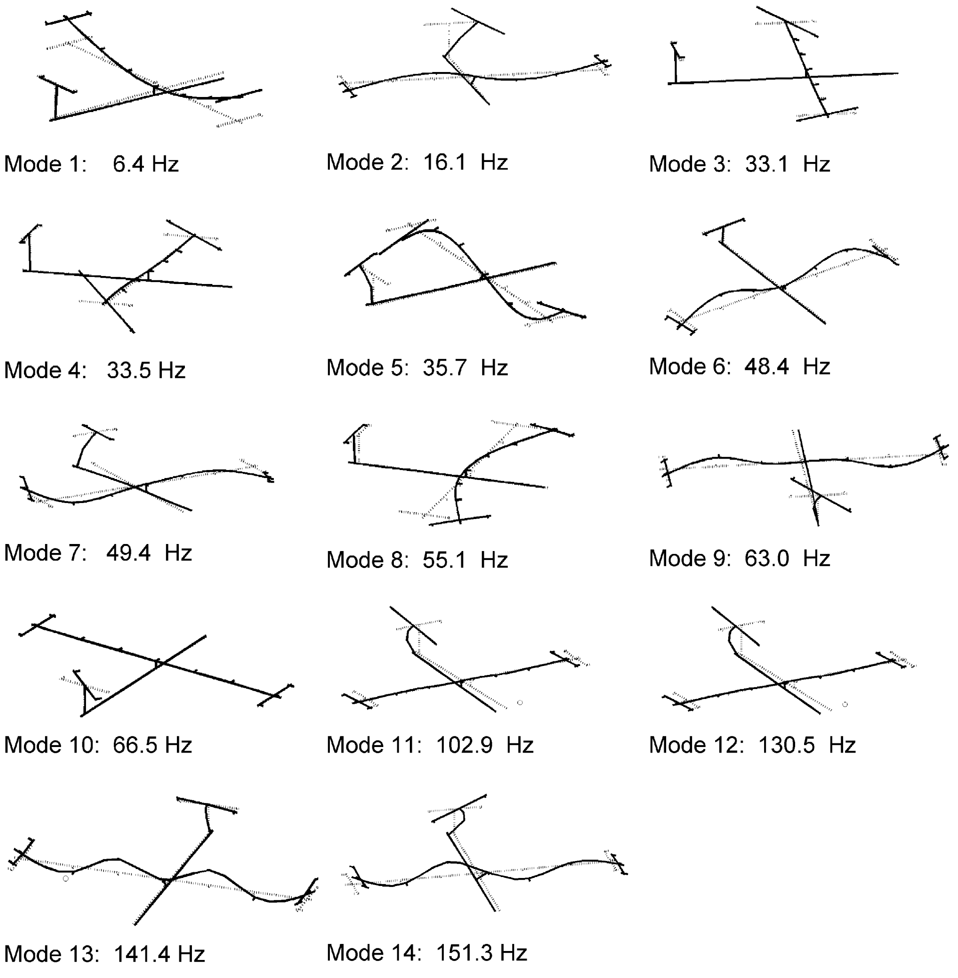

Figure 4. Modes measured by phase resonance testing.

TABLE 1

Comparison of modal data from four test institutes

Mode no.	Natural frequency (Hz)		Difference (%)	MAC (%)	Natural frequency (Hz)		MAC (%)
	DLR	U_MAN	DLR/ U_MAN	DLR/ U_MAN	IC	UWS	UWS/ DLR
1	6.38	6.51	−2.00	100	6.54	6.55	99
2	16.10	16.37	−1.65	100	16.55	16.61	98
3	33.13	33.44	−0.93	96	34.86	34.88	98
4	33.53	33.97	−1.30	99	35.30	35.36	97
5	35.65	36.17	−1.44	99	36.53	36.71	97
6	48.38	49.41	−2.08	100	49.81	50.09	99
7	49.43	50.20	−1.53	99	50.63	50.72	99
8	55.08	55.61	−0.95	100	56.39	56.44	100
9	63.04	64.04	−1.56	99	64.96	65.14	99
10	66.50	69.39	−4.16	100	69.82	69.64	98
11	102.90					105.47	99
12	130.54					134.68	91
13	141.38					145.87	99
14	151.32					155.82	99

can only be roughly estimated but must be taken into consideration during model updating. Table 1 also shows the natural frequencies of the unmodified structure taken just before the modifications were implemented. These measurements, taken approximately 5 years after the original tests, show a small systematic increase in the natural frequencies. The mode shapes correlation between the new and the original measurements is very good, with all but one (91% for mode 12) of the *MAC* values being better than 96% although a comparison of the FRFs from the early U_MAN and the new IC/UWS measurements of the unmodified structure reveals some differences in particular for the lateral x- and y-responses as shown in Fig. 3 for three selected dofs.

3. THE MODIFIED STRUCTURE

Two modifications were designed. The first consisted of two brass cylinders that were bolted onto the outside edge of the tail, close to node 301. Each cylinder was 60 mm in diameter and 20 mm in height, with a mass of 0.92 kg. The second modification consisted of replacing the cylinder of mass 150 g at node 112 on the wing, with one with a mass of 724 g. The brass cylinder had a diameter of 59 mm and a height of 30 mm, and its centre was located at the same place as the original cylinder.

Table 2 shows the natural frequencies of the structure with the two modifications, and clearly shows that the modifications significantly change the structure's dynamics. Table 3 shows the *MAC* matrix between the mode shapes of the unmodified structure and with modification 1. Clearly, the mode order has changed significantly. Modes 3–5 are very close in frequency and the modification has caused significant interaction of these modes. Table 4 shows the *MAC* matrix between the mode shapes of the unmodified structure and with modification 2. For this modification, the mode order for the lower modes is almost unchanged, although modes 6 and 7 have swapped order.

TABLE 2

Natural frequency (Hz) of the modified GARTEUR structure

Mode no.	Natural frequency (Hz) Modification 1 (IC)	Natural frequency (Hz) Modification 2 (UWS)
1	6.54	6.31
2	13.94	16.43
3	32.36	27.28
4	35.09	35.36
5	35.52	36.46
6	38.10	48.92
7	48.68	50.01
8	50.17	54.34
9	56.46	64.54
10	58.16	69.55
Modification	920 g mass added at node 301	150 g mass at node 112 replaced by 724 g

TABLE 3

The MAC values (%) between the modes of the unmodified structure and with modification 1

Mode no. after modification 1	1	2	3	4	5	6	7	8	9	10
Mode no. of unmodified structure	1	2	5	4	4	10	7	6	8	9
MAC	99	98	45	70	67	94	44	95	76	82

TABLE 4

The MAC values (%) between the modes of the unmodified structure and with modification 2

Mode no. after modification 2	1	2	3	4	5	6	7	8	9	10
Mode no. of unmodified structure	1	2	3	4	5	7	6	8	9	10
MAC	78	97	85	93	93	89	100	80	99	100

4. SUMMARY OF MODEL VALIDATION RESULTS

In this paragraph, we summarise the results obtained by the seven participants. A detailed description of the individual results is presented by the participants in their contributions following this summary paper. The methods applied may be classified according to

(1) the type of test/analysis residuals,
(2) the type of FE model (beam or shell elements) and
(3) the type and number of updating parameters. Table 5 gives an overview of the parameter sets chosen by the participants and highlights the very different choices made.

The following philosophy to generate a validated model was used by the participants:

- Step 1, Parameter selection: Either sensitivity and/or automated error location procedures were used in conjunction with physical reasoning.

Table 5

Type of updating parameters

Participant no.	Fuselage	Wing	Right wing	Left wing	Vertical wing off-set from fuselage	Horizontal wing off-set fuselage	Wing/fuselage connection
1	EI_{min}	EI_{min}, EI_{max}, ρ	GI_{tors}	GI_{tors}	Yes	Yes	
2	EI_{min}	EI_{min}, EI_{max}, GI_{tors}, ρ					Beam GI_{tors}
3	EI_{min}	EI_{min}, EI_{max}, GI_{tors}, ρ					Beam GI_{tors}
4		EI_{min}, GI_{tors}				Yes	Beam GI_{tors}
5		2 param., one for aluminium one for sandwich part					4 springs
6	E,G	E,G					E,G steelplate
7			I_{min},I_{max},I_{tors}	I_{min},I_{max},I_{tors}			

Participant no.	Vertical tail	Horiz. tail plane	Horiz. tail plane/ vert. tail	Vert. tail/ fuselage	Right drum	Left drum	Overall mass density	Residual type	FE-model type
1	EI_{min}			Off-set				f, Φ	Beam
2	EI_{min}	EI_{min}						f, af	Beam
3	EI_{min}							f	Beam
4			2 generic element parameters	Off-set				f	Beam
5	E		Mass		2 masses	2 masses		CRE	Plate
6	E,G				Mass	Mass	ρ	f, MAC	Plate
7	I_{min}							f, MAC	Beam

$EI_{min,max}$: minimum, maximum bending stiffness, GI_{tors}: torsional stiffness, E: Young's modulus, G: shear modulus, ρ: mass density, f: natural frequencies, af: antiresonances, Φ: mode shapes, CRE: constitutive relation error, MAC: modal assurance criterion.

- Step 2, Initial model tuning: This step included updating the parameters of the initial model of the unmodified structure and to check if the model was capable of predicting the experimental modal data within the active frequency range (criterion 1). Some participants extended the checks with respect to the passive frequency range (criterion 2).
- Step 3: Check of prediction capability concerning the modified structures (criterion 4).

It was quite difficult to make general conclusions on what are the best results and how they were achieved. Comparing the results with respect to a more or less arbitrary limit of $\pm 2\%$ for the frequency errors and 90% for the *MAC* values in view of the test data variability of the investigated test structure showed that all participants fulfilled this requirement for criteria 1, 2 and 4, if it was applied to the mean values of the frequency errors and the *MAC*s averaged over 10 modes. Of course, looking at the mean values alone disguises the outliers. As expected the prediction accuracy was generally lower for the passive (criterion 2) than for the active frequency range (criterion 1) with some remarkable outliers larger than 5% on the predicted natural frequencies in the passive range.

In the following, we concentrate on criterion 4, which checks the capability of the updated models to predict the effects of the structural modifications on the modal behaviour. Looking for the number of frequency errors exceeding 2% (and not distinguishing between the results for the tail and wing modifications) showed two participants with one outlier, two participants with three outlier and two participants with six outlier. Comparing the results with respect to the number of *MAC* values falling below 80% (a value estimated as reasonably accurate with respect to a worst case prediction and the actual measurement variability) gives no clear picture, one participant has one outlier, three have two and three have three outlier.

The worst case scenario was also investigated. The maximum frequency errors, as predicted by the seven participants, were: −2.22, −3.10, 3.36, −3.60, 5.85, 5.87 and −6.5%. For the worst case *MAC* values, the ranking was 79, 79, 70.8, 67, 64.8, 53.7 and 48.6%. These values and the mean values of the frequency errors and the *MAC* values averaged over 10 modes are shown in Tables 6–9. Figure 5 visualises the numbers given in these tables and should help the reader to make his own judgement. A criterion accepting

TABLE 6

Per cent frequency error, tail modification (mean of absolute errors for modes 1–10)

	Participant no.						
Mode	1	2	3	4	5	6	7
1	−1.07	−0.77	−1.02	0.04	0.75	0.60	5.06
2	1.08	−1.85	−2.93	−0.48	−0.62	2.70	−1.36
3	−1.95	−2.37	3.36	−1.92	−1.06	−0.80	−5.69
4	−0.14	0.56	0.54	0.49	0.27	1.70	−0.03
5	0.08	0.79	0.59	−0.14	0.41	1.70	−0.34
6	−1.57	−3.10	−3.08	−0.33	−6.50	2.50	0.94
7	−1.19	−1.42	−2.42	−1.01	−0.27	0.70	−2.16
8	0.60	−0.17	0.61	−0.49	1.09	1.60	0.26
9	0.39	−0.11	−0.30	−0.12	0.52	−3.60	2.44
10	−0.09	−1.09	−2.50	−1.83	−1.23	1.70	−1.98
11	−1.11	−0.49	−3.91	—	−7.00	—	−4.27
Mean	0.82	1.22	1.74	0.69	1.27	1.76	2.03

TABLE 7

Percent frequency error after wing modification (mean of absolute errors for modes 1–10)

	Participant no.						
Mode	1	2	3	4	5	6	7
1	−2.22	−2.23	−2.53	−0.81	2.18	−1.20	−5.87
2	1.28	−0.51	−1.29	0.88	0.86	−1.80	−0.18
3	1.80	1.36	1.40	5.85	−0.87	1.00	3.15
4	0.00	−1.47	−1.56	−0.80	0.37	1.90	−0.22
5	0.82	1.14	0.89	−0.83	−0.41	−0.40	−0.77
6	−1.35	−2.02	−1.95	−0.01	2.54	−0.30	−0.10
7	0.84	0.08	0.86	−0.59	0.90	−1.30	0.14
8	0.09	−0.40	−0.33	0.09	0.94	0.90	2.19
9	1.02	−0.66	−0.84	−0.20	1.48	2.00	−1.32
10	1.67	−0.46	−0.45	0.09	−0.84	−0.50	1.06
11	−0.46	−3.06	−4.98	—	—	—	−7.50
Mean	1.11	1.03	1.21	1.02	1.14	1.13	1.50

TABLE 8

MAC after tail modification (mean values for modes 1–10)

	Participant no.						
Mode	1	2	3	4	5	6	7
1	99.70	99.52	99.51	100.00	99.00	99.20	99.60
2	99.40	98.84	98.78	99.00	99.00	92.80	98.60
3	99.20	94.32	95.91	99.00	99.00	92.60	97.30
4	79.00	95.81	95.99	96.00	82.00	94.90	77.10
5	99.40	70.77	71.94	67.00	99.00	93.70	99.10
6	97.10	98.88	99.07	97.00	99.00	64.80	98.00
7	92.50	88.97	84.37	92.00	90.00	83.40	88.80
8	98.60	94.77	94.90	98.00	99.00	96.30	98.80
9	93.50	91.65	53.70	86.00	95.00	85.20	65.90
10	97.70	94.72	69.12	93.00	98.00	85.50	84.60
11	96.40	96.34	93.01	—	90.00	—	92.20
Mean	95.61	92.82	86.32	92.70	95.90	88.84	90.78

one or two outliers (only one or two frequency errors higher than 2%, and one or two *MAC* values below 80%) was met by four participants. It should be kept in mind that the above limits were used here to make the results distinguishable, and they may be different for other applications. For example, higher frequency errors are generally accepted for the upper modes than for the lower modes.

It is interesting to note that five participants have found about the same *MAC* of 79% for mode no.1 for the wing modification (one has 96.7% and one has only 48.6%) which means that this mode shape did not change significantly after introducing the modification into the model. This *MAC* value is about the same as that obtained from comparing the measured unmodified/modified structure mode no. 1 ($MAC = 78\%$ according to Table 4). So it seems as if a good prediction for mode no. 1 was more difficult to obtain in the case of the wing modification than in the case of the tail modification, where all participants

Table 9

MAC after wing modification (mean values for modes 1–10)

	Participant no.						
Mode	1	2	3	4	5	6	7
1	79.00	78.83	78.85	79.00	78.92	96.70	48.60
2	96.00	92.74	92.46	94.00	95.29	92.30	93.10
3	97.40	95.22	95.19	96.00	97.33	86.80	96.60
4	91.20	94.67	94.69	94.00	88.41	83.30	91.80
5	90.60	88.76	88.12	94.00	88.61	89.80	88.70
6	97.40	93.53	92.68	89.00	97.31	84.40	90.90
7	96.90	91.73	91.86	94.00	97.01	79.80	95.50
8	96.90	84.14	83.61	99.00	96.82	76.10	97.40
9	93.00	87.20	86.36	93.00	92.74	87.30	91.30
10	95.50	95.41	95.41	95.00	95.36	95.70	95.40
11	92.00	89.65	88.04	—	93.69	—	89.90
Mean	93.39	90.22	89.92	92.70	93.60	87.22	88.93

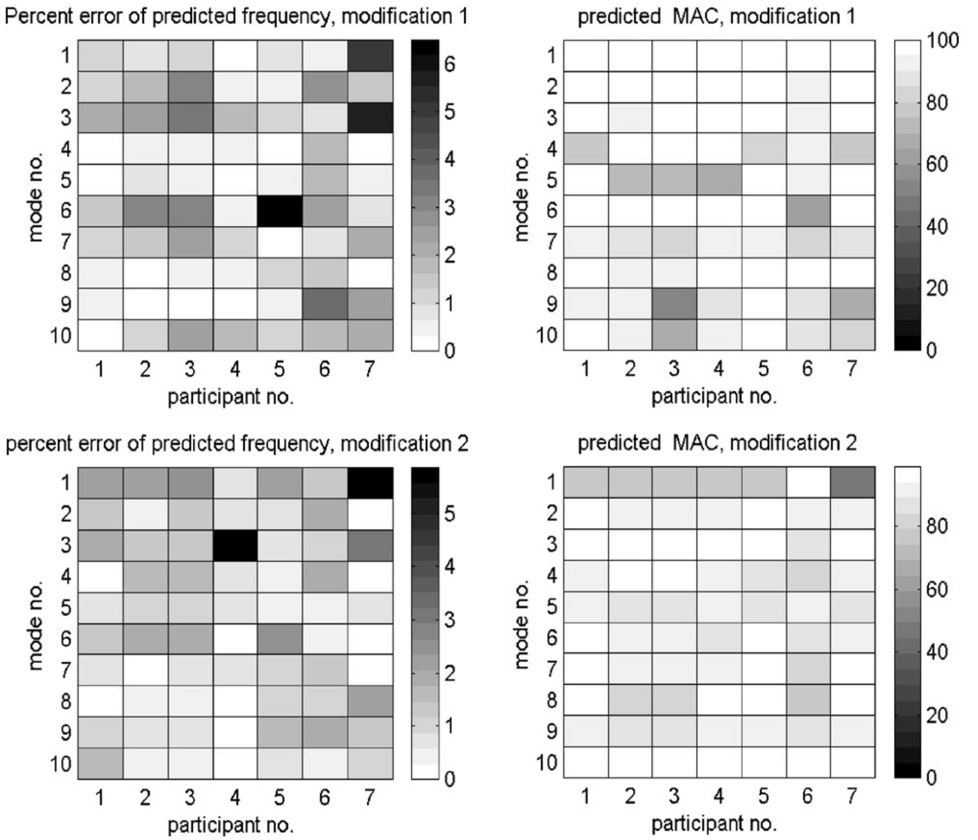

Figure 5. Predicted frequency errors and *MAC* values after modification 1 (tail) and 2 (wing).

predicted MAC values of more than 99% which, of course, is not astonishing since mode 1 was not significantly affected by the tail modification ($MAC = 99\%$ according to Table 3).

Since all the participants used different residuals for their objective function to be minimised, and different types of structural idealisation (beam or plate elements), it is difficult to make recommendations on what residual is best. The choice of parameter sets given in Table 5 reveals a wide variety. All participants used different wing stiffness parameters, all but one used the vertical tail stiffness, and all but one used parameters to represent the wing/fuselage connection. Obviously, it was necessary to carefully select the parameters describing the connections, particularly when beam models were used. The most important issue was to find an appropriate parameter set. With this requirement fulfilled, good prediction results can be expected, even with the simple eigenfrequency residual. Automated computational procedures help in the selection process but must be supplemented by physical reasoning. Looking at the great variety of parameters, it is clear that even though the parameters are called physical or geometric (like Young's modulus or a beam off-set) they must be interpreted as non-unique equivalent parameters describing lumped stiffness and mass properties. It is interesting to note that good prediction capabilities of the updated models have been achieved in many different ways. Thus, it is believed that the knowledge resulting from this benchmark study is of more general importance and should stimulate practical applications for the present updating technology.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the following organisations: ONERA (France), for making available the GARTEUR test-bed, DLR (Germany), and University of Manchester (U.K.) for making available the original test data and the Imperial College of Science, Technology and Medicine, London (U.K.) and the University of Wales Swansea (U.K.) for re-testing the structure and making available the test data.

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